As an example, we shall examine the random process $\varphi_1(x)$, whose correlation function has the form

$$D_0 \cos\frac{\pi x_1}{l}\cos\frac{\pi x_2}{l}, \quad D_0 = \text{const}.$$

The parameters of the problem (1)-(3) are as follows

$$\varphi_2(x) \equiv 0, \quad \alpha_{2i} \equiv 0, \; l_0 = 0.19\cdot 10^{-6}, \; i = 1, 2.$$

Figure 1 shows the change in the mean-square temperature as a function of coordinates and time, where the characteristic size l_0 is chosen according to [2].

If τ_r equals the ratio of the Maxwellian relaxation time and some function of the relaxation coefficient [3], then the working equations proposed can be used without any changes to determine the corresponding probabilistic characteristics of the random multidimensional temperature fields of bounded, homogeneous, isotropic bodies. In addition, the summation must be understood as summation with respect to the natural increasing order of the Laplacian operator of the corresponding multidimensional problem.

LITERATURE CITED

1. J. Akhmadi, "Heat conduction in solids with initial conditions in the form of a random function," Mech. Eng., Ser. C, Heat Transfer, No. 4, 96 (1974), pp. 44-47.
2. K. V. Lakusta and Yu. A. Timofeev, "Some estimates of the limits of applicability of the hyperbolic heat-conduction equation," Inzh.-Fiz. Zh., 37, No. 2, 366-370 (1979).
3. V. A. Bubnov, "Notes on wave equations in the theory of heat conduction," in: Problem of Heat And Mass Transfer [in Russian], Nauka i Tekhnika, Minsk (1976), pp. 168-175.

ONE-DIMENSIONAL MODEL OF HEAT TRANSFER IN CRYOGENIC VACUUM-SHIELD THERMAL INSULATION WITH RADIANT HEAT SOURCES

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The heat-transfer problem in an insulation consisting of layers which receive heat from external source through radiation is numerically solved in the one-dimensional approximation.

It is well known that many characteristics of modern cryogenic devices are determined by the thermal properties of the vacuum-shield thermal insulation stacks. Accordingly, more efficient new compositions of such stacks are being developed in many countries. At the same time, there is still sufficient margin for improvement left in existing vacuum-shield insulation, inasmuch as the effective thermal conductivity of these stacks in cryogenic devices is at least 1.5-3 times higher than that of the best laboratory specimens [1, 2].

According to an earlier analysis [3], one of the causes of this worsening is the presence of numerous channels running across a stack of vacuum-shield thermal insulation (gaps around the neck of the vessel, around the support rods, around the cooled object, between insulation layers, etc.) and letting hot radiation pass directly to the cold layers of the stack. Since that analysis [3] was a semiquantitative one and hardly any other studies on this subject were ever made, the authors have developed a model for calculating the heat transfer through the layers of vacuum-shield thermal insulation and taking into account the interaction of these layers with external radiant heat sources.

The main difficulties in the mathematical formulation of such a problem arise due to the intricate dependence of the thermal flux entering the insulation layers on the law of tempera-

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ture variation along the channel, this law in turn depending largely on the power of the radiant heat sources. As a consequence, in the stack of a vacuum-shield thermal insulation with channels penetrating it one can find an either two- or three-dimensional temperature field interacting with a compound flux of radiant heat along the boundaries of those channels. Even in a simplified form can such a problem be solved with large expenditures of time on only the most modern computers with large memories. We therefore have studied the possibility of reducing the problem to a one-dimensional one, considering that in a vacuum-shield thermal insulation stack the longitudinal thermal conductivity is 3-4 orders of magnitude higher than the transverse thermal conductivity. As a consequence, the temperature gradients along the insulation layers must be much smaller than those across them so that the layers can, in the first approximation, be regarded as longitudinally isothermal ones. The validity range of this assumption can be estimated on the basis of the ratio $(\delta/l)\sqrt{\lambda_{\parallel}/\lambda} \gtrsim 10$ [4]. One more dimension of the thermal field is eliminated, viz., even thick stacks of vacuum-shield thermal insulation can be regarded as being plane and their thickness small in comparison with the diameter of the cryogenic vessel.

For final touch-up of the calculation method a vacuum-shield thermal insulation stack with a hole of the simplest form was selected, a straight circular cylinder, such a configuration being most suitable for easy experimental verification. The method was then applied to long narrow channels appearing in vacuum-shield thermal insulation stacks with loose contacts between individual layers.

Rather than the simplest Fourier equation

$$\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) = 0 \quad (1)$$

a Fourier equation with volume sources of heat

$$\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) = q_v(x,\ T) \quad (2)$$

must be used for describing one-dimensional heat transfer in a vacuum-shield thermal insulation stack with a hole and given boundary temperatures

$$x = 0 \quad T = T_0 = \text{const}, \quad (3)$$

$$x = \delta \quad T = T_\delta = \text{const}. \quad (4)$$

The quantity q_v is assumed to be uniformly distributed over the surface of each layer, since the longitudinal thermal conductivity in the stack has been assumed to be infinitely high. The temperature dependence of the thermal conductivity of a vacuum-shield thermal insulation is taken from another study [5].

In order to determine the quantity $q_v(x, T)$, it is necessary to examine the radiant heat flowing through the hole to the wall of the stack. For the purpose of solving this problem, we treat the cylindrical channel as a closed system consisting of N separate surfaces at a constant temperature T_i with diffuse reflection. The density of the resultant radiant flux in such a system is described by the system of equations [6]

$$q_i = \frac{\varepsilon_i}{1-\varepsilon_i}[\sigma T_i^4 - B_i],\ i = 1,\ 2,\ \ldots,\ N. \quad (5)$$

For determining the distribution B_i we have the system of linear integral equations

$$B_i = \varepsilon_i\sigma T_i^4 + (1-\varepsilon_i)\sum_{j=1}^{N}\int_{S_j} B_j d\varphi_{di-dj}. \quad (6)$$

The angular coefficients between elementary surfaces are taken from another study [7]. Having thus determined q_i, we find the resultant radiant flux for each surface

$$Q_i = q_i S_i. \quad (7)$$

From the value of Q_i at the lateral surfaces of the channel, we obtain the magnitude of the volume source of radiant heat

$$q_v(x_i,\ T_i) = \frac{Q_i}{S_{\text{ins}}\Delta x}. \quad (8)$$

It has already been mentioned that, because of the interdependence of the temperature field in a vacuum-shield thermal insulation and the radiative heat transfer along a channel, the trend of the function $q_V = q_V(x, T)$ cannot be stipulated beforehand. Problem (2)-(7) will, therefore, be solved by the iteration method with q_V first assumed to be zero and the temperature field according to Eq. (1) taken as the first approximation. On the basis of this temperature profile, we solve Eqs. (5)-(7) of radiative heat transfer in the channel and obtain the second approximation for the distribution of source q_V across the thickness of the vacuum-shield thermal insulation. Then, using the function $q_V = q_V(x)$, we proceed to solve Eq. (2). The number of successive approximations depends on the requirement as to how small the difference between the values of the thermal flux through the vacuum-shield thermal insulation obtained in two consecutive approximations should be (in our calculations this difference was stipulated to be smaller than 0.01%).

For solving the quasilinear equation (2) by the finite-difference method, we used the Kirchhoff substitution $\Phi=\int_{T_0}^{T}\lambda(T)\,dT$, which made it possible to reduce this equation to $\partial^2\Phi/\partial x^2 = q_V$. The system of linear integral equations (6) was reduced to a system of algebraic equations by application of the trapezoidal rule to the integrals on the right-hand sides.

It must be emphasized that as the radiant heat flux q_V (and thus the resultant Q_i) increases, there also increases the difference between the sought actual temperature profile $T_S = T_S(x)$ in the insulation stack and its first approximation $T_1 = T_1(x)$ obtained from Eq. (1). Calculations have revealed that in the range of q_V values (based on T_1 values) more than 1.5-2 times higher than the specific thermal flux through a vacuum-shield thermal insulation $q_1 = \lambda dT_1/dx$, the iteration process becomes divergent. It was found that convergence in this range can be achieved only with the initial temperature distribution stipulated as accurately as within 0.01°K of the sought temperature profile T_S. For a better comprehension of the problem, let us note that the difference $T_S - T_1$ can exceed 20-30°K. For obtaining an initial temperature distribution T_b with the required accuracy, therefore, we have developed the following procedure.

From the temperature distribution without a heat source $T_1 = T_1(x)$ according to the solution to the system of equations (2)-(7) we determined the temperature distribution $T_2 = T_2(x)$. The values of function T_2 at all points exceeded those of functions T_1 and T_S (Fig. 1). Therefore, the sought solution T_S lay between functions T_1 and T_2 and could be found by the following algorithm.

On the basis of profiles T_1 and T_2 we constructed profile T_3 as follows. We let $T_3 = T_2$ at the points along the x-coordinate where the values of T_2 did not exceed the values of T_1 by more than $\alpha_i = k/(n - i + 1)$, n denoting the number of layers into which the insulation stack had been subdivided, i denoting the consecutive number of a layer counted from the warm boundary wall, and the constant K assumed to be equal to 2. At the other points (where $T_{2,i} > T_{1,i} + \alpha_i$) we used the values $T_{1,i} + 0.9\alpha_i$ for T_3. Then from the temperature distribution $T_3 = T_3(x)$, according to Eqs. (2)-(7), we found a new profile $T_4 = T_4(x)$. Profiles T_3 and T_4, obtained by this procedure, lay closer to the sought solution than did the initial profiles T_1 and T_2.

Applying this procedure again to profiles T_3 and T_4, we found a new distribution T_5 from which again, on the basis of Eqs. (2)-(7), we found a distribution T_6. A new profile T_7 was then again found from distributions T_5 and T_6. It is to be noted that as profiles T_5, T_7, or analogous ones are being constructed, there may appear points at which $T_{4,i} < T_{3,i}$ or $T_{6,i} < T_{5,i}$, etc. For such points in the course of finding the distributions T_5, T_7, and others we let $T_{5,i} = T_{3,i}$, $T_{7,i} = T_{5,i}$, etc. The operation of plotting the profiles T_5, T_7, ... is to be repeated so long until the difference between two adjacent temperature distributions becomes smaller than α_i.

Along one of these profiles we followed through 2-4 iteration steps toward solving the system of equations (2)-(7) and, subsequently, from the last two functions found we selected the profile with the lower temperature. The operation of plotting profiles of the T_3, T_5, etc. kind was repeated several times, but with a smaller α_i. The constant K was decreased from 2 to 0.1 in these calculations. We again repeated the cycle of plotting profiles of the T_3 kind until the difference between them became smaller than α_i. Along one of these profiles we followed through 2-4 iterations in the system of equations (2)-(7), which already ensured

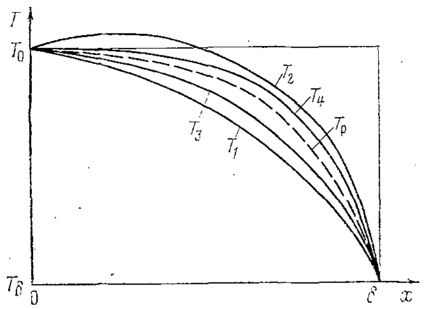

Fig. 1

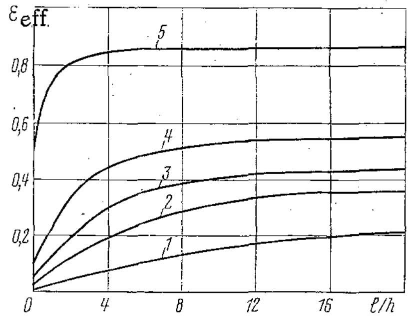

Fig. 2

Fig. 1. Successive temperature profiles in the insulation obtained by the iteration process for the solution of the problem with high-power radiant heat sources.

Fig. 2. Effective emissivity at the face of the vacuum-shield thermal insulation stack as a function of the relative elongation of the gap between shield layers, for various values of their emissivity: 1) $\varepsilon = 0.01$; 2) 0.03; 3) 0.05; 4) 0.1; 5) 0.5.

that the temperature differed by less than 0.01°K and the thermal flux through the insulation stack differed by less than 0.01% from one iteration to the next.

The described method can also yield the solution in the case of high-power radiant heat sources. The machine time on a model M-222 computer then increases from 12-15 to 40-60 min.

For calculating by this method the heat transfer in a vacuum-shield thermal insulation with radiative heat transfer in the channel taken into account, one needs to have data on the emissivity of the cylindrical lateral channel walls. The absorption by these walls is determined by the optical characteristics of the gaps between adjacent layers of the vacuum-shield thermal insulation. For determining the emissivity ε_{eff} at the fact of an interlayer gap, we have calculated the effective emissivity of an infinitely wide strip of a thickness equal to the gap thickness h between layers. For this purpose, Eqs. (5)-(6) of radiative heat transfer were solved again by that method [6]. In those calculations all walls were assumed to have the same emissivity ε (equal to the emissivity ε of the shield layers) uniform along the entire channel.

The reflection by the walls was assumed to be diffuse and the effect of the packing material on the heat transfer in a gap was disregarded. In that study the emissivity was varied from 0.01 to 0.1 and the channel elongation was varied from 1 to 100. The results of the calculations are shown in Fig. 2. It is to be noted that ε_{eff} has been calculated here for the $\varepsilon = 0.1$-0.9 range, for which data are already available [6]. The complete agreement between our results and those in [6] served as one criterion for verifying the correctness of our program of calculating the radiative heat transfer.

With a low emissivity of the layers (0.01-0.05), the maximum emissivity of a gap face ε_{eff} is attained at relative channel elongations $l/h = 20$-30. This emissivity ε_{eff} is then 10-25 times higher than the emissivity of the insulation layers. Accordingly, the face of the vacuum-shield thermal insulation stack has an emissivity almost within the 0.3-0.5 range.

It must be taken into consideration that the presence of a packing material between insulation layers can additionally increase the emissivity of the stack face, it can also make the maximum ε_{eff} attainable at smaller channel elongations. Since the emissivity of the lateral channel walls in a vacuum-shield thermal insulation is high, it is necessary to devise methods of its reduction. One of these methods is coating these walls with one layer of aluminized polyethylene terephthalate film which has an emissivity within the 0.03-0.05 range. Such values of ε were, accordingly, used in subsequent calculations of the heat transfer in vacuum-shield insulation.

As a parameter characterizing the efficiency of an insulation stack, we introduce its degradation coefficient

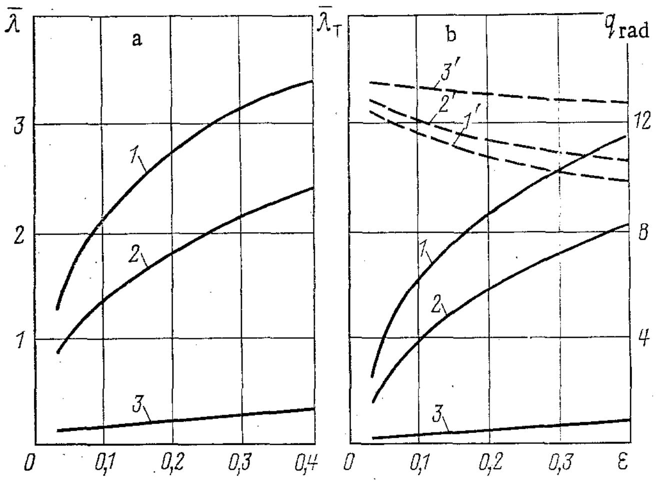

Fig. 3. Dependence of the components of the thermal flux reaching a cryogenic vessel on the emissivity of the face of the vacuum-shield thermal insulation stack with a thickness δ: 1, 1') 0.11 m, 2, 2') 0.075 m, 3, 3') 0.0165 m; (a) total thermal flux through the insulation $\bar{\lambda}$; (b) components of the thermal flux reaching the bottom base of the channel by radiation q_{rad} and by conduction through the insulation (thermal conductivity λ_T).

$$\bar{\lambda} = \bar{\lambda}_T + \bar{\lambda}_{rad} = \frac{Q_T - Q_0}{Q_0} + \frac{Q_{rad}}{Q_0}. \tag{9}$$

In this study was examined the dependences of the thermal flux through a vacuum-shield thermal insulation stack on the boundary temperatures, the emissivity of the stack face, and the stack thickness. The lateral surface of the hole was for these calculations subdivided into narrow cylindrical elements and the temperature of each was assumed to be constant. Also examined was the dependence of the thermal flux on the number of those elements. As their number was increased from 15 to 30, the thermal flux through the insulation stack with given parameters was found to have changed by not more than 1-8%. Therefore, it was not deemed worthwhile to increase the number of elements further. Calculations were also made assuming a uniform effective radiant flux, and then taking into account the nonuniformity of the thermal flux at both lower and upper channel bases. The results did not differ by more than 2% so that in subsequent calculations the thermal flux at both channel bases could be assumed to be uniform. Calculations were made for an insulation stack with an area of 1.5 m^2, having a hole 0.11 m in diameter and emissivity of 1.0 at the upper channel face and of 0.03 at the lower channel base.

The calculations have revealed that increasing the temperature of the warm layer of the insulation will degrade the efficiency of the stack noticeably more than increasing the temperature of its cold layer. This trend becomes particularly pronounced as the emissivity of the stack face is increased.

Let us examine the dependence of the insulation efficiency on its thickness. The graph in Fig. 3 depicts $\bar{\lambda}$, $\bar{\lambda}_T$, and q_{rad} as functions of the emissivity of the face of the vacuum-shield thermal insulation stack for various thicknesses of the latter. The dash lines correspond to q_{rad}. It is evident here that as the emissivity of the lateral surface of the channel increases, the thermal flux impinging on its base decreases and becomes an insignificant part of the total thermal flux through thick stacks with a high emissivity. At the same time, however, the thermal flux through the insulation increases and this causes the thermal conductivity of the insulation $\bar{\lambda}_T$ as well as the total heat transfer through the stack ($\bar{\lambda}$) to become worse. For this reason, thinner stacks of vacuum-shield thermal insulation are relatively more efficient under conditions of strong nonuniformity.

In vessels with vacuum-shield thermal insulation there often exists gaps between individual stacks, around the neck of the vessel, at the supports, etc. In this connection, the graph in Fig. 4 shows the results of calculations of the thermal characteristics made for a vacuum-shield thermal insulation stack with a long slot in the form of a rectangular channel. The emissivity of the stack face was assumed to be 0.4 here. The graph depicts (for various

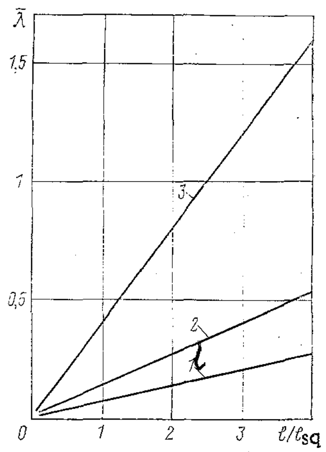

Fig. 4. Dependence of the coefficient of insulation efficiency degradation on the relative length l/l_{sq} of a rectungular slot of width h: 1) 0.0005 m; 2) 0.001 m; 3) 0.002 m.

slot widths h) the coefficient of insulation degradation $\bar{\lambda}$ on the slot length l, the latter referred to the side l_{sq} of a square with an area equal to that of the vacuum-shield thermal insulation stack. The data here indicate that through slots of a length equal to the side of a square vacuum-shield thermal insulation stack already increase the thermal flux through the insulation from 8 to 40%.

In the process of calculations for a vacuum-shield thermal insulation with slots and holes, we also determined the radiant thermal flux Q_s reaching the face of the insulation stack. The relation between Q_s and the thermal flux Q_0 conducted by the insulation was defined by the ratio $k_0 = (Q_T - Q_0)/Q_s$. These calculations have revealed that k_0 varies from 0.5 to 0.9, which agrees with the conclusions of our earlier study [3]. The thus-calculated thermal flux Q_s was then compared with the thermal flux Q_{s0} reaching the face of the insulation stack in the zeroth-order approximation of the temperature distribution, as in the case when interaction of the radiant thermal flux with the thermal flux through the insulation stack is disregarded and the total thermal flux is assumed to be the additive sum of both components. It has been found that Q_{s0} is 1.5-4 times larger than Q_s. Therefore, calculating the radiative heat transfer without taking into account its interaction with the heat transfer inside the insulation and then evaluating the latter heat transfer on this basis will result in a substantial error in the determination of the thermal flux through the insulation.

In conclusion, we summarize as follows. We have constructed a one-dimensional approximate model of heat transfer through a vacuum-shield thermal insulation stack under conditions of radiative heat transfer to the stack face. We have also developed a procedure for solving problems which involve high-power radiant heat sources.

The study has revealed that increasing the emissivity of the stack face can noticeably (by a factor of 2-3) degrade the efficiency of the insulation. The radiant thermal flux reaching the bottom base of a channel decreases as the emissivity ε of the stack face is increased. It has been demonstrated that coating the lateral surface of such a channel with one layer of polyethylene terephthalate film having a low emissivity will improve the efficiency of the insulation stack. It has also been established that in the case of a highly nonhomogeneous insulation stack a thinner one will be relatively more efficient. Calculation of the radiative heat transfer in such a channel without taking into account its interaction with the heat transfer inside the insulation stack has been found to result in large errors (up to 50-100%).

The proposed thermal model and calculation method can be used for calculating the thermal flux through an insulation, with the interaction of vessel necks, brackets, and supports with the vacuum-shield thermal insulation stack taken into account. Even the effect of contacts between insulation layers can be taken into account. The results of an experimental verification of this calculation method and the conclusions based on it will be presented in the next report.

NOTATION

T, temperature; x, coordinate in the direction normal to the stack; δ, stack thickness; λ, thermal conductivity of the vacuum-shield thermal insulation along the x-coordinate; q_V, amount of heat released in a unit volume of vacuum-shield thermal insulation as a result of the incidence of radiation on the face of the shield layers; ε, emissivity; B, density of effective radiant flux; σ, Stefan–Boltzmann constant; N, number of surfaces; S_{ins}, surface of a vacuum-shield thermal insulation stack; $d\varphi_{di-dj}$, angular coefficient between elementary areas i and j of a surface; Q_0, thermal flux through the insulation without a hole; Q_T, thermal flux through the insulation with a hole; Q_{rad}, thermal flux through the insulation with radiative heat transfer to the bottom base of the hole; Q_S, thermal flux reaching the insulation with a hole; Q_{rad}, thermal flux through the insulation with radiative heat transfer to the bottom base of the hole; Q_S, thermal flux reaching the stack face by radiative heat transfer through the hole (channel); Δx, thickness of the i-th insulation layer including one or more shields; ε_{eff}, effective emissivity of the gap face; q_{rad}, amount of heat reaching a unit area of the bottom base of the hole; T_0 and T_0, boundary temperatures; l, length of the gap between layers of the insulation stack and the length of the vacuum-shield thermal insulation stack; h, width of the gap between layers of the insulation stack; and $\lambda_{||}$, longitudinal thermal conductivity of the vacuum-shield thermal insulation.

LITERATURE CITED

1. M. G. Kaganer, Thermal Insulation in Low-Temperature Techniques [in Russian], Mashinostroenie, Moscow (1966).
2. G. A. Bell, T. C. Nast, and R. K. Wedel, "Thermal performance of multilayer insulation applied to small cryogenic tankage," Adv. Cryogen. Eng., 22, 272-282 (1977).
3. V. F. Getmanets, R. S. Mikhal'chenko, and V. D. Vakulenko, "Nonadditivity of thermal fluxes through 'thermal bridges' and vacuum-shield thermal insulation of cryogenic devices," in: Heat Transfer at Low Temperatures [in Russian], Naukova Dumka, Kiev (1979), pp. 120-130.
4. I. A. Paivanas, O. F. Roberts, and I. J. Wang, "Multishielding in advanced superinsulation technique," Adv. Cryogen. Eng., 10, 197 (1965).
5. R. S. Mikhal'chenko, N. P. Pershin, E. I. Shchirov, and N. I. Gerasimenko, "Experimental study of the dependence of the thermal properties of multilayer insulation on the temperature, the vacuum level, and on the stacking density," in: Problems of Hydrodynamics and Heat Transfer in Cryogenic Systems [in Russian], No. 3, Izd. Fiz.-Tekh. Inst. Nizk. Temp. Akad. Nauk Ukr. SSR, Kharkov (1973), pp. 100-105.
6. E. M. Sparrow and R. D. Cess, Radiation Heat Transfer, Brooks-Cole (1969).
7. R. Siegel and J. R. Howell, Thermal Heat Transfer, McGraw-Hill (1972).